

LDPC DECODER AND METHOD THEREOF

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CROSS REFERENCE TO RELATED APPLICATIONS

The present invention claims priority under 35 U.S.C. §119 (e) from U.S. provisional application serial no. 60/214781, entitled "Address Generator for LDPC Encoder and Decoder and Method Thereof," filed June 28, 2000, the contents of which are incorporated herein by reference.

The present invention is a continuation-in-part application of application Serial No. 09/559186, entitled "Multi-Mode Iterative Detector" and filed on April 27, 2000, the contents of which are incorporated herein by reference,

The present invention is related to the following commonly-assigned, copending applications:

"LDPC Encoder and Method Thereof", filed on even date and assigned application Serial No. _____ (Attorney Docket No. MP0064), the contents of which are incorporated herein by reference,

"Address Generator for LDPC Encoder and Decoder and Method Thereof" filed on even date and assigned application Serial No. _____ (Attorney Docket No. MP0063), the contents of which are incorporated herein by reference, and

"Parity Check Matrix and Method of Forming Thereof", filed on even date and assigned application Serial No. _____ (Attorney Docket No. MP0069), the contents of which are incorporated herein by reference.

BACKGROUND OF THE INVENTION

Field of the Invention

The present invention relates generally to a linear block decoder in a data transmission system. More particularly, the present invention relates to a low density parity-check code (LDPC) decoder a read channel in a disk drive system.

Description of the Related Art

Fig. 1 illustrates a conventional digital data transmission system. As shown therein, a digital data transmission system comprises a transmitting section 300 for transmitting user data to receiver 500 via communication channel 401.

The operation of transmission section 300 will now be explained. Prior to processing by transmitting section 300, input or user data maybe encoded with an error correcting code, such as the Reed/Solomon code, or run length limited encoded (RLL) or a combination thereof by encoder 302. The encoded output encoder 302 is then interleaved by deinterleaver 308 for input to linear block code encoder 304 which generates parity data in a known manner utilizing linear block codes. One example of a linear block code is a low-density parity-check code (LDPC) which is discussed by Robert G. Gallager in *Low-Density Parity-Check Codes*, 1963, M.I.T. Press and by Zining Wu in *Coding and Iterative Detection For Magnetic Recording Channels*, 2000, Kluwer Academic Publishers, the contents of each of which are incorporated in their entirety by reference. Deinterleaver 308 permutes the data so that the same data is reordered before encoding by linear block code encoder 304. By permuting or redistributing the data, interleaver 306 attempts to reduce the number of nearest neighbors of small distance error events. User data at the output of encoder 302 is referred to as being in the channel domain; that is the order in which data is transmitted through the channel. The order of data processed by deinterleaver 308 is referred to as being in the linear block code domain. The parity data from linear block code encoder 304 is combined with the data encoded by encoder 302 by multiplexer 306 for input to channel transmitter 310.

Transmitter 310 transmits the combined user and parity data from multiplexer 306 typically as an analog signal over communication channel 401 in the channel domain. Communication channel 401 may include any wireless, wire, optical and the like

communication medium. Receiver 500 comprises a front-end circuit 502 comprising analog to digital and equalization circuits. The digital signal front-end circuit 502 is input to soft channel decoder 504, which provides probability information of the detected data. Soft channel decoder may be implemented by a Soft Viterbi Detector or the like. The output of the soft channel decoder 504, which is in the channel domain, is converted into the linear block code domain by deinterleaver 510. Deinterleaver 510 is constructed similarly to deinterleaver 308. Soft linear block code decoder 506 utilizes this information and the parity bits to decode the received data. One output of soft linear block code decoder 506 is fed back to soft channel decoder 504 via interleaver 512, which converts data in the linear block code domain to the channel domain. Deinterleaver 512 is constructed to perform the reverse operations of deinterleaver 510. Soft channel decoder 504 and soft linear block code decoder 506 operate in an iterative manner to decode the detected data.

The other output of soft linear block code decoder 506 is converted from the linear block domain to the channel domain by interleaver 514. Deinterleaver 514 is constructed similarly to interleaver 512. The output of interleaver 514 is passed on for further processing to decoder 508. Decoder 508 is implemented to perform the reverse operations of encoder 302.

Summary of the Invention

According to a first aspect of the present invention, a method of decoding low-density parity-check codes is provided comprising the steps of (a) calculating $llrR_m$, for each parity check equation, at iteration $i-1$, in response to step c, (b) decision aided equalizing, at iteration i , in response to step a; and (c) calculating $llrQ_{lm}$, for each parity check equation, at iteration i in response to step b. $llrQ_{lm}$ represents information from bit node l to equation node m , one for each connection, and $llrR_m$ represents information from equation node m to bit node l , one for each connection.

According to a second aspect of the present invention, a method of decoding low-density parity-check codes is provided further comprising the steps of (d) updating, for each parity check equation, the smallest $llrQ_{lm}$ calculated in step c, at iteration i , (e) updating, for each parity check equation, the second smallest $llrQ_{lm}$ calculated in step c, at iteration i ; and (f) updating, for each parity check equation, an overall sign of $llrQ_{lm}$, calculated in step c, at iteration i . Step a is responsive to step d, step e and step f, and step c is responsive to step a.

According to a third aspect of the present invention, step a is calculated as follows:

$$llrR_{ml}^{i-1} = \begin{cases} -s_m^{i-1} \cdot \text{sign}(llrQ^{i-1}_{lm}) \cdot \min 1^{i-1}_m, & \text{if } l \neq l_m^{i-1} \\ -s_m^{i-1} \cdot \text{sign}(llrQ^{i-1}_{lm}) \cdot \min 2^{i-1}_m, & \text{otherwise.} \end{cases}$$

Step c is calculated as follows:

$$llrQ_{lm}^i = llrP_l^i + \sum_{m' \neq m} llrR_{m'l}^{i-1}$$

According to a fourth aspect of the present invention, step c is initialized with soft channel information.

According to a fifth aspect of the present invention, the method further comprises the steps of (g) converting $|llrAPP_k|$ information into hard information $b_{c,k}$, (h) calculating an equation syndrome for each parity check equation, s_i . (i) summing each of equation syndrome in step h, (j) outputting the hard information if the sum in step i is equal to zero; (k) repeating steps a-c if the sum in step i is not equal to zero, (l) determining if i is less than a first predetermined value; (m) repeating steps a-c, if in step l, i is less than the first predetermined value; (n) determining if the sum in step i is less than a second predetermined value; and (o) outputting the hard information if i is at least the first predetermined and the sum in step i is less than the second predetermined value.

According to a sixth aspect of the present invention, the method further comprises the steps of (p) for each data k , determining if $|llrAPP_k|$ is less than a third predetermined value; (q) for each data k , outputting hard information $b_{c,k}$ if $|llrAPP_k|$

is at least the third predetermined value; and (r) for each data k , outputting soft channel information $b_{s,k}$ if $|llrAPP_k|$ is less than the third predetermined value.

According to a seventh aspect of the present invention, the method further comprises the steps of (s) for each data k , determining if a corresponding parity check equation is violated, (t) for each data k , outputting hard information $b_{c,k}$, if the corresponding parity check equation is not violated, (u) for each data k , determining if $|llrAPP_k|$ is less than a third predetermined value, (v) for each data k , outputting hard information $b_{c,k}$ if $|llrAPP_k|$ is at least the third predetermined value, and (w) for each data k , outputting soft channel information $b_{s,k}$ if $|llrAPP_k|$ is less than the third predetermined value.

According to an eighth aspect of the present invention, a computer program for decoding low-density parity-check codes is provided comprising the steps of (a) calculating $llrR_{ml}$, for each parity check equation, at iteration $i-1$, in response to step c, (b) decision aided equalizing, at iteration i , in response to step a; and (c) calculating $llrQ_{lm}$, for each parity check equation, at iteration i in response to step b. $llrQ_{lm}$ represents information from bit node l to equation node m , one for each connection, and $llrR_{ml}$ represents information from equation node m to bit node l , one for each connection.

According to a ninth aspect of the present invention, a computer program for decoding low-density parity-check codes is provided further comprising the steps of (d) updating, for each parity check equation, the smallest $llrQ_{lm}$ calculated in step c, at iteration i , (e) updating, for each parity check equation, the second smallest $llrQ_{lm}$ calculated in step c, at iteration i ; and (f) updating, for each parity check equation, an overall sign of $llrQ_{lm}$, calculated in step c, at iteration i . Step a is responsive to step d, step e and step f, and step c is responsive to step a.

According to a tenth aspect of the present invention, step a is calculated as follows:

$$llrR_{ml}^{i-1} = \begin{cases} -s_m^{i-1} \cdot \text{sign}(llrQ_{lm}^{i-1}) \cdot \min 1^{i-1}_m, & \text{if } l \neq l_m^{i-1} \\ -s_m^{i-1} \cdot \text{sign}(llrQ_{lm}^{i-1}) \cdot \min 2^{i-1}_m, & \text{otherwise.} \end{cases}$$

Step c is calculated as follows:

$$llrQ_{lm}^i = llrP_l^i + \sum_{m' \neq m} llrR_{m'l}^{i-1}$$

According to an eleventh aspect of the present invention, step c is initialized with soft channel information.

According to a twelfth aspect of the present invention, the computer program further comprises the steps of (g) converting $|llrAPP_k|$ information into hard information $b_{c,k}$, (h) calculating an equation syndrome for each parity check equation, s_i , (i) summing each of equation syndrome in step h, (j) outputting the hard information if the sum in step i is equal to zero; (k) repeating steps a-c if the sum in step i is not equal to zero, (l) determining if i is less than a first predetermined value; (m) repeating steps a-c, if in step l, i is less than the first predetermined value; (n) determining if the sum in step i is less than a second predetermined value; and (o) outputting the hard information if i is at least the first predetermined and the sum in step i is less than the second predetermined value.

According to a thirteenth aspect of the present invention, the computer program further comprises the steps of (p) for each data k , determining if $|llrAPP_k|$ is less than a third predetermined value; (q) for each data k , outputting hard information $b_{c,k}$ if $|llrAPP_k|$ is at least the third predetermined value; and (r) for each data k , outputting soft channel information $b_{s,k}$ if $|llrAPP_k|$ is less than the third predetermined value.

According to a fourteenth aspect of the present invention, the computer program further comprises the steps of (s) for each data k , determining if a corresponding parity check equation is violated, (t) for each data k , outputting hard information $b_{c,k}$, if the corresponding parity check equation is not violated, (u) for each data k , determining if

$|\text{llrAPP}_k|$ is less than a third predetermined value, (v) for each data k , outputting hard information $b_{c,k}$ if $|\text{llrAPP}_k|$ is at least the third predetermined value, and (w) for each data k , outputting soft channel information $b_{s,k}$ if $|\text{llrAPP}_k|$ is less than the third predetermined value.

5 According to a fifteenth aspect of the present invention, a decoder for decoding low-density parity-check codes is provided comprising first calculating means for calculating llrR_{ml} , for each parity check equation, at iteration $i-1$, decision aided equalizing means for equalizing llrR_{ml} , at iteration i , in response to the first calculating means, and second calculating means for calculating llrQ_{lm} , for each parity
10 check equation, at iteration i in response to the decision aided equalizing means. llrQ_{lm} represents information from bit node l to equation node m , one for each connection, and llrR_{ml} represents information from equation node m to bit node l , one for each connection, and the first calculating means is responsive to the second calculating means.

15 According to a sixteenth aspect of the present invention, The decoder, further comprises memory means for storing for each parity check equation, the smallest llrQ_{lm} calculated by the second calculating means, at iteration i , the second smallest llrQ_{lm} calculated by the second calculating means, at iteration i ; and an overall sign of llrQ_{lm} , calculated by the second calculating means, at iteration i . The first calculating
20 means is responsive to the memory means, and the second calculating means is responsive to the first calculating means.

According to a seventeenth aspect of the present invention, the first calculating means calculates as follows:

$$\text{llrR}_{ml}^{i-1} = \begin{cases} -s_m^{i-1} \cdot \text{sign}(\text{llrQ}_{lm}^{i-1}) \cdot \min 1^{i-1}_m, & \text{if } l \neq l_m^{i-1} \\ -s_m^{i-1} \cdot \text{sign}(\text{llrQ}_{lm}^{i-1}) \cdot \min 2^{i-1}_m, & \text{otherwise.} \end{cases}$$

25 The second calculating means calculates as follows:

$$llrQ_{lm}^i = llrP_i^i + \sum_{m' \neq m} llrR_{m'i}^{i-1}$$

According to an eighteenth aspect of the present invention, the second calculating means is initialized with soft channel information.

According to a nineteenth aspect of the present invention, the decoder, further
5 comprises an address generating means for providing indices of each of the parity check equations.

According to a twentieth aspect of the present invention, the second calculating means provides an index of the smallest $llrQ_{lm}$.

According to a twenty-first aspect of the present invention, multiplying means is provided for scaling $llrR_{mi}$, calculated by the first calculating means.

According to a twenty-second aspect of the present invention, the decoder further comprising slicing means for converting $|llrAPP_k|$ information into hard information $b_{c,k}$; equation vector means for calculating an equation syndrome for each parity check equation s_i ; summing means for summing each equation syndrome calculated by the equation vector means. The hard information is output if the sum by the summing means is equal to zero, and the calculations by the first and second calculating means are repeated if the sum summed by the summing means is not equal to zero.

According to a twenty-third aspect of the present invention, the decoder further
20 comprises first threshold means for determining if i is less than a first predetermined value, and second threshold means for determining if the sum by the summing means is less than a second predetermined value. The calculations by the first and second calculating means are repeated if i is less than the first predetermined value as determined by the first threshold means, and the hard information is output if i is at
25 least the first predetermined and the sum by the summing means is less than the second predetermined value.

According to a twenty-fourth aspect of the present invention, the decoder further comprises third threshold means for determining for each data k if $|llrAPP_k|$ is less than a third predetermined value. For each data k , hard information $b_{c,k}$ is output if
30 $|llrAPP_k|$ is at least the third predetermined value as determined by the third threshold means; and for each data k , soft channel information $b_{s,k}$ is output if

$|\text{llrAPP}_k|$ is less than the third predetermined value as determined by the third threshold means.

According to a twenty-fifth aspect of the present invention, the decoder further comprises judging means for determining for each data k if a corresponding parity check equation is violated and third threshold means for determining for each data k if $|\text{llrAPP}_k|$ is less than a third predetermined value. For each data k , hard information $b_{c,k}$, is output if the corresponding parity check equation is not violated as determined by the judging means, for each data k , hard information $b_{c,k}$ is output if $|\text{llrAPP}_k|$ is at least the third predetermined value as determined by the third threshold means; and wherein for each data k , soft channel information $b_{s,k}$ is output if $|\text{llrAPP}_k|$ is less than the third predetermined value as determined by the third threshold means.

According to a twenty-sixth aspect of the present invention, a decoder for decoding low-density parity-check codes is provided comprising a first calculator to calculate llrR_{ml} , for each parity check equation, at iteration $i-1$, a decision aided equalizer to equalize llrR_{ml} , at iteration i , in response to the first calculator; and a second calculator to calculate llrQ_{lm} , for each parity check equation, at iteration i in response to the decision aided equalizer. llrQ_{lm} represents information from bit node l to equation node m , one for each connection, and llrR_{ml} represents information from equation node m to bit node l , one for each connection. The first calculator is responsive to the second calculator.

According to a twenty-seventh aspect of the present invention, the decoder further comprises a memory to store for each parity check equation, the smallest llrQ_{lm} calculated by the second calculator, at iteration i ; the second smallest llrQ_{lm} calculated by the second calculator, at iteration i ; and an overall sign of llrQ_{lm} , calculated by the second calculator, at iteration i . The first calculator is responsive to the memory, and the second calculator is responsive to the first calculator.

According to a twenty-eighth aspect of the present invention, the first calculator calculates as follows:

$$llrR_{ml}^{i-1} = \begin{cases} -s_m^{i-1} \cdot \text{sign}(llrQ_{lm}^{i-1}) \cdot \min 1^{i-1}_m, & \text{if } l \neq l_m^{i-1} \\ -s_m^{i-1} \cdot \text{sign}(llrQ_{lm}^{i-1}) \cdot \min 2^{i-1}_m, & \text{otherwise.} \end{cases}$$

The second calculator calculates as follows:

$$llrQ_{lm}^i = llrP_l^i + \sum_{m' \neq m} llrR_{m'l}^{i-1}$$

According to a twenty-ninth aspect of the present invention, the second calculator is initialized with soft channel information.

According to a thirtieth aspect of the present invention, the decoder further comprises an address generator for providing indices of each of the parity check equations.

According to a thirty-first aspect of the present invention, the second calculator provides an index of the smallest $llrQ_{lm}$.

According to a thirty-second aspect of the present invention, the decoder further comprises a multiplier to scale $llrR_{ml}$, calculated by the first calculator.

According to a thirty-third aspect of the present invention, the decoder further comprises a slicer to convert $|llrAPP_k|$ information into hard information $b_{c,k,i}$, and an equation vector circuit to calculate an equation syndrome for each parity check equation, s_i .

According to a thirty-fourth aspect of the present invention, the decoder further comprises a summer to sum each equation syndrome calculated by the equation vector circuit. The hard information is output if the sum summed by the summer i is equal to zero, and the calculations by the first and second calculator are repeated if the sum summed by the summer is not equal to zero.

According to a thirty-fifth aspect of the present invention, the decoder further comprises a first threshold detector to determine if i is less than a first predetermined value; and a second threshold detector to determine if the sum by the summer is less than a second predetermined value. The calculations by the first and second calculator

are repeated if i is less than the first predetermined value as determined by the first threshold detector; and the hard information is output if i is at least the first predetermined and the sum by the summing means is less than the second predetermined value.

5 According to a thirty-sixth aspect of the present invention, the decoder further comprises a third threshold detector to determine for each data k if $|\text{llrAPP}_k|$ is less than a third predetermined value. For each data k , hard information $b_{c,k}$ is output if $|\text{llrAPP}_k|$ is at least the third predetermined value as determined by the third threshold detector; and for each data k , soft channel information $b_{s,k}$ is output if
10 $|\text{llrAPP}_k|$ is less than the third predetermined value as determined by the third threshold detector.

According to a thirty-seventh aspect of the present invention, the decoder further comprises a judging circuit to determine for each data k if a corresponding parity check equation is violated, and a third threshold detector to determine for each
15 data k if $|\text{llrAPP}_k|$ is less than a third predetermined value. For each data k , hard information $b_{c,k}$ is output, if the corresponding parity check equation is not violated as determined by the judging circuit, and for each data k , outputting hard information $b_{c,k}$ if $|\text{llrAPP}_k|$ is at least the third predetermined value as determined by the third threshold detector; and for each data k , soft channel information $b_{s,k}$ is output if
20 $|\text{llrAPP}_k|$ is less than the third predetermined value as determined by the third threshold detector.

Other objects and attainments together with a fuller understanding of the invention will become apparent and appreciated by referring to the following description and claims taken in conjunction with the accompanying drawings.

25 Brief Description of the Drawings

In the drawings wherein like reference symbols refer to like parts.

Fig. 1 is a block diagram of a conventional data transmission system;

Fig. 2 is a block diagram of a data transmission system in accordance with the present invention;

30 Fig. 3 is an example of a parity check matrix in accordance with the present invention;

Fig. 4 is a block diagram of a memory arrangement of the decoder in accordance with the present invention;

Fig. 5 is a block diagram of the decoder in accordance with the present invention;

5 Fig. 6 is an example of a factor graph;

Fig. 7 is an example of a factor graph illustrating the calculation of $llrQ_{lm}$;

Fig. 8 is an example of a factor graph illustrating the calculation of $llrR_{ml}$;

Fig. 9 is a graph of a function for calculating a soft decision;

Fig. 10 is a block diagram of a decision circuit of the decoder of Fig. 5;

Fig. 11 is a block diagram of an alternate decision circuit of the decoder of Fig. 5;

Fig. 12 is a factor graph of the decoding procedure in accordance with the present invention;

Fig. 13 is a flow diagram of the decoding procedure in accordance with the present invention;

Fig. 14 is a flow chart of the decision procedure of the decision circuit of Fig. 10;

Fig. 15 is a flow chart of the decision procedure of the alternate decision circuit of Fig. 11;

Fig. 16 is a block diagram of a conventional decoding implementation; and

Fig. 17 is a block diagram of a decoding implementation in accordance with the present invention.

Description of the Preferred Embodiments

Reference is now made to Fig. 2, which is a block diagram of a data transmission system in accordance with the present invention. In general as shown therein, a digital data transmission system comprises a transmitting section 300' for transmitting user data to receiver 500' via communication channel 401. The inventors have observed that a linear block code encoder is not dependent on a position of a bit interleaved. Rather the linear block code encoder only requires a list of equations for a given bit. In other words, there is no need to process the data in the order defined by the interleaver, instead data may be processed in the same order as it is written to the channel. This can be accomplished by incorporating an address generator to provide

an address of the appropriate equation of the linear block code encoder. This principle can be similarly applied to the soft linear block decoder. As a result, deinterleaver 308 of the conventional system is now replaced by address generator 328, and deinterleaver 510 is now replaced by address generator 530. Accordingly, there is no requirement for the physical interleaving of data in the receiver 500', since the data remains in the same order as the order of bits of data in the channel throughout this system. The order of bits of data transmitted through the channel is referred to as the channel domain.

The operation of transmission section 300' will now be explained. Prior to processing by transmitting section 300', as in the conventional system, input or user data maybe encoded with an error correcting code, such as the Reed/Solomon code, or run length limited encoded (RLL) or a combination thereof by encoder 302. Addresses for the parity equations of linear block code encoder 304 are generated by address generator 328 in accordance with an index of the bits of data, the index being determined by address generator 328. Address generator 328 is responsive to counter 730 under the control of controller 740. Controller 740 synchronizes counter 730 to the output of encoder 302 so that counter 730 can provide a count of the number of bits in a codeword output by encoder 302 and a count of the number of codewords. In the preferred embodiment the data block size is 5000 bits.

Referring again to Fig. 2, linear block code encoder 304 utilizes the user data and address from address generator 328 to provide the parity bits to multiplexer 306. Address generator 328 is described in more detail in commonly assigned, copending patent application entitled "Address Generator for LDPC Encoder and Decoder and Method Thereof" filed on even date and assigned application Serial No. _____ (Attorney Docket No. MP0063), the contents of which are incorporated herein by reference. Linear block code encoder 304 is preferably implemented as a low-density parity-check code (LDPC) encoder as described in commonly assigned, copending patent application entitled "LDPC Encoder and Method Thereof", filed on even date and assigned application Serial No. _____ (Attorney Docket No. MP0064), the contents of which are incorporated herein by reference. The parity data from linear block code encoder 304 is combined with the data encoded by encoder 302 by multiplexer 306 for input to channel transmitter 310. In the preferred embodiment, the combined data consists of series of a pair parity bits followed by 34 bits of user data. This constraint is established by RLL constraint encoder 302.

Transmitter 310 transmits the combined user and parity data from multiplexer 306 typically as an analog signal over communication channel 401 in the channel domain. Communication channel 401 may include any wireless, wire, optical, magnetic and the like.

5 Receiver 500' comprises an analog to front-end circuit 502 comprising analog to digital and equalization circuits. The digital signal from front-end circuit 502 is input to soft channel decoder 504, which provides soft or probabilistic information of the detected data to soft linear block decoder 506. Soft channel decoder may be implemented as a Soft Viterbi Detector or the like, and address generator 530 may be constructed similarly as address generator 328 in transmission section 300'. In the preferred the Soft Viterbi Detector may be implemented with a Soft-Output Viterbi Algorithm which is described in J. Hagenauer and P. Hoeher: "A Viterbi algorithm with soft-decision outputs and its applications", Proc. IEEE GLOBECOM '90, Dallas, Tex., pp. 47.1.1-47.1.7, November 1989, the contents of which are incorporated by reference.

10 The soft information output by soft channel decoder 504 remains in the channel domain and is decoded by soft linear block code decoder 506, in accordance with the address of the parity equations generated by address generator 530. Address generator 530 is responsive to counter 735 under the control of controller 745. Controller 745 synchronizes counter 735 to the output of soft channel decoder 504 so that counter 735 can provide a count of the number of bits in a codeword output by soft channel decoder 504 and a count of the number of codewords.

15 Soft linear block code decoder 506 operates in combination with soft channel decoder 504 and address generator 530 in an iterative fashion. Soft linear block code decoder is preferably implemented as a low-density parity-check code (LDPC) decoder, as described in detail hereinbelow. It is noted that since the soft information from soft channel decoder 504 to soft linear block code decoder 506 are both in the channel domain, thus as noted above, there is no need for any interleavers or deinterleavers in receiver 500'.

20 After the iterative process has completed, the output of soft linear block code decoder 506 is passed on for further processing to decoder 508. Decoder 508 is implemented to perform the reverse operations of encoder 302 or correct for any data errors.

Prior to discussing the construction and operation of the LPDC decoder, reference is now made to Fig. 3 for an explanation of the parity check matrix. Fig. 3 shows only one example of a parity check matrix. Of course, other parity check matrices are contemplated. The preferred matrix is 222 rows (or equations) by 5402 columns, which comprises 220 linearly independent rows (where $5402 = 73 \times 74$). The matrix can be divided into three tiers of equations having 73, 74 and 75 equations, respectively. The set of independent rows can be obtained by canceling the last row of the second tier and third tier, namely the 147th row and the 222nd row. As shown in Fig. 3, the following table shows the values of the elements in the matrix:

Tier	i^{th} position	i^{th} position
1	1 if $r = i(\text{mod}73)$	0 if $r \neq i(\text{mod}73)$
2	1 if $r = i(\text{mod}74)$	0 if $r \neq i(\text{mod}74)$
3	1 if $r = i(\text{mod}75)$	0 if $r \neq i(\text{mod}75)$

A matrix having 5402 columns can process a maximum LDPC codeword of 5402 bits. Of course, as will be appreciated by one of ordinary skill in the art, the matrix may be truncated to accommodate a smaller block, however the matrix must be at least 222×4366 which is dependent on the constraint of encoder 302. This constraint is for example a RLL constraint. The preferred matrix contains no cycles, since a matrix having cycles has degraded performance that degrades significantly. With the first tier only, the parity check matrix has a $D_{\min}=2$; by adding the second tier, the parity check matrix has a $D_{\min}=4$; and by adding the third tier, the parity check matrix has a $D_{\min}=6$. A further description of the parity check matrix is provided in commonly assigned, copending application entitled, "Parity Check Matrix and Method of Designing Thereof", filed on even date and assigned application Serial No. _____ (Attorney Docket No. MP0069), the contents of which are incorporated herein by reference.

The LDPC decoder is preferably implemented using the sum-product algorithm, which is described by Zining Wu in *Coding and Iterative Detection For Magnetic Recording Channels*, 2000, Kluwer Academic Publishers, the contents of each of which are incorporated in its entirety by reference.

A linear code is a set of codewords, \mathbf{x} , satisfying the matrix equation (1)

$$\mathbf{H} \cdot \mathbf{x} = 0 \quad (1),$$

where \mathbf{H} is an $M \times L$ matrix, and \mathbf{x} is a $1 \times L$ vector.

The parity check matrix for an LDPC encoder/decoder is sparse, that is a small portion of the elements being one, all other elements being zero. An example of a parity check matrix is shown in equation 2 that corresponds to the factor graph shown in Fig. 6.

$$H = \begin{bmatrix} 1011 \\ 0111 \end{bmatrix} \quad (2)$$

which defines the following parity-check equations for the codeword \mathbf{x}

$$x_1 + x_3 + x_4 = 0$$

$$x_2 + x_3 + x_4 = 0$$

Fig. 6 is a factor graph of the parity matrix of equation 2. The factor graph contains two types of nodes the bit node (e.g. b_1 , b_2 , b_3 , and b_4) and the check nodes (e.g. e_1 , e_2). Each bit node corresponds to a bit in the codeword, and each check node represents a parity-check equation (i.e., a row in the parity check matrix \mathbf{H}). Hence, the factor graph for an LDPC code with an $M \times L$ parity check matrix \mathbf{H} contains M check nodes and L bit nodes. An edge between a check node and a bit node exists if and only if the bit participates in the parity-check equation represented by the check node. The factor graph shown in Fig. 3 is "bipartite" in which the nodes can be separated to two groups, namely check nodes and bit nodes. Connections are allowed only between nodes in different groups.

A cycle in a factor graph refers to a finite set of connected edges that start and end at the same node. The bold lines in Fig. 6 represent a cycle length of four. As can be appreciated by one of ordinary skill in the art, four is the shortest cycle length a parity check matrix can have.

In the sum-product decoding algorithm, there are four types of soft information circulating in the decoding process:

1. $llrP_i$: The soft information from channel APP (*a posteriori* probability) decoder or soft channel decoder 504 for the first iteration or from decision aided equalization circuit 856 for subsequent iterations, one for each bit.

2. $llrQ_{lm}$: The information from bit node l to equation node m , one for each connection.

3. $llrR_{ml}$: The information from equation node m to bit node l , one for each connection.

5 4. $llrAPP_l$: the overall soft information after each iteration, one for each bit.

Each iteration performs three steps of operations (superscripts denote iteration number):

1. Each bit calculates the information $llrQ$ that it passes to the connecting equations, which is the sum of the extrinsic information $llrR$ from the previous iteration with the channel information $llrP$ from the current iteration, excluding the extrinsic information from the same equation, i.e.:

$$llrQ_{lm}^i = llrP_l^i + \sum_{m' \neq m} llrR_{m'l}^{i-1} \quad (3)$$

2 Each check equation (each row) calculates the extrinsic information for each involved bit. The "LLRXOR" ($\Sigma \oplus$) denotes the LLR operation discussed below.

$$llrR_{ml}^i = \sum_{l' \neq l} llrQ_{l'm}^i \quad (4)$$

20 3. Each bit calculates the overall soft information by summing up all the $llrR_{ml}$'s and the $llrP_l$

$$llrAPP_l^i = llrP_l^i + \sum_m llrR_{ml}^i \quad (5)$$

The "LLRXOR" operation is defined as:

$$\begin{aligned} 25 \quad LLRXOR(y,z) &= \log(e^y + e^z) - \log(e^{y+z} + e^0) \quad (6) \\ &= \max(y,z) + \log(1 + e^{-|y-z|}) - \max(x+y, 0) - \log(1 + e^{-|y+z|}) \end{aligned}$$

where $\log(1 + e^{-|x|})$ can be implemented with a look-up table. Such an operation requires 5 additions, 2 absolute values and two table look-ups from a look-up table.

30 This procedure is referred to as the "Sum Product" (SP) algorithm.

The "LLRXOR" operation can be further simplified by eliminating the table look-ups, i.e. omitting the $\log(1+e^{-|a|})$ term in equation (6). The equation is simplified as follows:

$$\text{LLRXOR}(y,z) = -\text{sign}(y)\text{sign}(z)\min(|y|, |z|) \quad (7).$$

As is apparent to one of ordinary skill in the art, equation (7) requires only absolute value and comparison operations. This operation is referred to as the "Signed Sum Product" (SSP) algorithm.

In the SSP algorithm, the magnitudes of the extrinsic information conveyed in each equation are determined by the two log-likelihood ratios with the smallest amplitudes. Other LLR's only contribute to the sign of the information, not to the amplitude. Fig. 7 shows the llrQ_{lm} 's passed from bits to an equation, and Fig. 8 shows the llrR_{ml} 's relayed by the equation.

A conventional implementation of SP or SSP requires L units of memory to store the llrP_t 's; L units of memory to store the llrAPP_t 's; ML units of memory to store the llrQ_{lm} 's; and ML units of memory to store the llrR_{ml} 's. Soft information is updated bit by bit within each iteration. These implementations are referred to as the "bit-based" implementations. As will be appreciated by one of ordinary skill in the art, the bit-based implementations require a large amount of memory and increased hardware complexity. Fig. 12 illustrates an implementation of SSP, which is generally performed in a serial manner. As shown therein, soft channel decoding is performed for the first iteration. Thereafter llrQ_{lm} and llrR_{ml} are sequentially calculated. For subsequent iterations i , decision aided equalization is then performed on the information. After the decision aided equalization iteration llrQ_{lm} is calculated in accordance with equation (3), and then llrR_{ml} is calculated in accordance with equation (4). The conventional process requires, for a 4000 bit codeword, approximately 4000 cycles for soft channel decoding, approximately 4000 cycles for calculating llrQ_{lm} , approximately 4000 cycles for calculating llrR_{ml} , and approximately 4000 cycles for performing decision aided equalization. The amount of time required by conventional methods degrades performance.

In accordance with the present invention, a pipelined architecture is proposed for the SSP circuit, as shown in Fig. 17. In accordance with the present invention, soft channel decoding is performed to initialize the SSP circuit, and for iteration i , llrQ_{lm}^i is calculated. At the same time llrR_{ml}^{i-1} for iteration $i-1$ is also calculated. Based on this result and llrAPP_t decision aided equalization is then performed. As can be appreciated by one of ordinary skill in the art, the pipelined architecture allows for

reduced pendency, resulting in higher throughput. In accordance with the present invention, each iteration takes about 4000 cycles for a 4000 bit codeword. According to another aspect of the present invention, an "equation-based" arrangement is proposed which implements the same algorithm with much less hardware complexity as described herein below.

The "equation-based" arrangement takes advantage of both the sparseness of the parity-check matrix and the simplicity in the SSP decoding. It is noted that the number of equations for each bit involved corresponds to the column weight of the parity check matrix, which in the preferred embodiment is three. Of course other appropriate column weights may be employed. According to equation (7), each equation node only passes two values to the connected bit nodes if signs of the soft information are not considered. Therefore, there is only a need to store two LLR magnitudes for each equation, and the signs for all the $llrQ_{lm}$'s. for each equation, the two smallest LLR's (min1 and min2), the position of min1 (i), and the sign of the equation(s); for each bit are stored. Additionally, three signs are stored corresponding to the signs of three $llrQ_{lm}$'s from the previously iteration. $llrR_{lm}$'s can be calculated from the stored information as follows:

$$llrR_{ml}^{i-1} = \begin{cases} -s_m^{i-1} \cdot \text{sign}(llrQ_{lm}^{i-1}) \cdot \text{min1}^{i-1}_m, & \text{if } l \neq l_m^{i-1} \\ -s_m^{i-1} \cdot \text{sign}(llrQ_{lm}^{i-1}) \cdot \text{min2}^{i-1}_m, & \text{otherwise.} \end{cases}$$

where $s_m^i = (-1) \prod_1 \text{sign}(llrQ_{lm}^i)$ is the sign of the mth equation

(8)

Referring to block diagram in Fig. 5, the corresponding flow chart in Fig. 13, and the corresponding factor graph in Fig. 12, the "equation-based" SSP circuit performs the following process:

1. The equation-based SSP circuit is initialized with the output of soft channel decoder 504 for the calculation of $llrP_i$, which is selected by multiplexer 854.

During subsequent iterations, multiplexer 854 selects the output of decision aided equalization circuit 856, as discussed hereinbelow.

2. At each time clock, address generator 804 calculates the indices of each equation for the current bit, and position index generator 802 calculates the position index thereof. Three $llrR_m$'s are calculated using Equation 8 by a calculating means or calculator or equation 1 LLR update circuit 816, equation 2 LLR update circuit 818, and equation 3 LLR update circuit 820.

3. Each bit calculates the information that it passes to the connecting equations by calculating means or calculator as follows,

$$llrQ_{lm}^i = llrP_l^i + \sum_{m' \neq m} llrR_{m'l}^{i-1} \quad (9)$$

The $llrR_m$'s are multiplied by a scaling factor by multipliers 836, 838 and 840 respectively to reduce circulation of any errors. The $llrR_m$'s are also delayed by delay circuits 832, 834, and 842, respectively, to synchronize with $llrP_l$.

4. Each bit stores the three signs of the $llrQ$'s in memory means or memory sections 708, 712 and 726 for equations 1, 2 and 3 respectively.

5. Each bit updates the equation information (min1, min2, i, and s) for the three equations involving the current bit by equation 1 update circuit 808, equation 2 update circuit 810, and equation 3 update circuit 830, respectively, in accordance with the following procedure:

If $|llrQ_{lm}^i| < min1_m^i$,
 $min1_m^i \rightarrow min2_m^i$
 $|llrQ_{lm}^i| \rightarrow min1_m^i$
 $i \rightarrow i_m$

 else if $|llrQ_{lm}^i| < min2_m^i$,
 $|llrQ_{lm}^i| \rightarrow min2_m^i$

 endif $s_m^i = -s_m^i \cdot \text{sgn}(llrQ_{lm}^i)$

5. Each bit calculates the overall soft information by summing up all the LLR_m 's, and LLR_P (via delay circuit 848) as an input to decision aided equalizer 856.

$$\text{LLRAPP}_i = \sum_m \text{LLR}_{mi} + \text{LLR}_P.$$

5 Log-likelihood ratios are converted to probability information by converter 850. Converter 850 implements the function shown in Fig. 9. In the preferred embodiment, the function is implemented as a piecewise linear approximation, shown as the dashed lines in Fig. 9. Decision aided equalizer 856 processes LLRAPP_i and samples stored in samples memory 864 from an equalizer (not shown) in front end 502. Decision aided equalization is described in more detail by Zining Wu in *Coding and Iterative Detection For Magnetic Recording Channels*, 2000, Kluwer Academic Publishers, the contents of which are incorporated in its entirety by reference.

Decision circuit 900, which is responsive to the output of summer 852, address generator 530 and soft channel decoder 504 determines when to terminate this iterative process, as is described in detail below.

Fig. 4 is a block diagram of a memory arrangement of the decoder in accordance with the present invention. As shown therein, memory or memory means 750 has a partition for each equation, and each equation has a partition for storing magnitude data, index data and sign data for iteration i and for iteration $i-1$. As used herein, the term partition refers either to a physical memory partition or a logical memory partition. Equation 1 update circuit 808, for each clock cycle, writes during iteration i to memory partition 806a or memory partition 806b i , and reads data previously stored from iteration $i-1$ from memory partition 806b or memory partition 806a. Multiplexer 902 selects the memory partition for write operations and multiplexer 904 selects the memory partition for read operations. Equation 1 LLR update circuit 816 reads data previously stored from iteration $i-1$ from memory partition 806b or memory partition 806a as selected by multiplexer 914. Equation 2 update circuit 810, equation 3 update circuit 830, equation 2 LLR update circuit 818, and equation 3 LLR update circuit 820 operate similarly.

Fig. 10 is a block diagram of decision circuit 900, and Fig. 14 is a flow chart thereof. Decision circuit 900 is responsive to the output from soft channel decoder 914, LLRAPP_i from summer or summing means 852 and address generator or address

generator means 530. Decision circuit 900 comprises memory 914 for storing hard decision data $b_{s,k}$ (for $k=1:5402$) from soft channel decoder 504, slicer or slicing means 888 for converting $|\text{llrAPP}_l|$ information into hard information $b_{c,k}$, memory 904 for storing $b_{c,k}$, a threshold detector 902 to determine if $|\text{llrAPP}_l|$ from summer 852 is less than a threshold value (t_2). Decision circuit or decision means 900 further comprises counter 920 for counting the number of iterations performed by the decoder and threshold detector 910 to determine if the number of iterations exceeds threshold value t_3 , which in the preferred embodiment is four. Equation vector circuit equation vector means 908 determines the equation syndromes, one for each equation, S_m , $m=1:222$ in which $S_m=0$ when equation i is satisfied or $S_m=1$ when equation m is violated. Summer 906 sums each of the S_m $m=1:222$.

The operation of decision circuit 900 will now be explained in conjunction with the flow chart of Fig. 14. After an iteration of iterative decoding, the equation syndrome is calculated by equal vector circuit 908 and summed by summer or summing means 906. Threshold circuit 912 determines if the sum of all $S_m = 0$, then a signal is sent to multiplexer 854 to output $b_{c,k}$ from memory 904. If the sum of all $S_m \neq 0$, then a determination is made by threshold 910 whether the number of iterations is less than threshold value t_3 . If the number of iterations is less than threshold value t_3 , the next iteration is performed. Otherwise threshold circuit 912 determines if the sum of all S_m is less than threshold t_1 . If so, then a signal is sent to multiplexer 854 to output $b_{c,k}$ from memory 904. Alternatively, the next step in the process is to determine if $|\text{llrAPP}_l|$ is less than threshold value (t_2), if so, multiplexer 854 selects $b_{s,k}$ to be output. If $|\text{llrAPP}_l|$ is not less than threshold value (t_2), multiplexer 854 selects $b_{c,k}$ to be output.

Fig. 11 is a block diagram of decision circuit 900' and Fig. 15 is a flow chart thereof. Decision circuit 900' is responsive to the output from soft channel decoder 914, llrAPP_l from summer 852 and address generator 530. Decision circuit 900' comprises memory 914 for storing hard decision data $b_{s,k}$ (for $k=1:5402$) from soft channel decoder 504, slicer 888 for converting $|\text{llrAPP}_l|$ information into hard information $b_{c,k}$, memory 904 for storing $b_{c,k}$, a threshold detector 902 to determine if $|\text{llrAPP}_l|$ from summer 852 is less than a threshold value (t_2). Decision circuit 900' further comprises counter 920 for counting the number of iterations performed by the decoder and threshold detector 910 to determine if the number of iterations exceeds threshold value t_3 , which in the preferred embodiment is four. Equation vector circuit 908 determines the equation syndromes, one for each equation, S_m , $i=1:222$ in which $S_m = 0$ when equation i

is satisfied or $S_m = 1$ when equation i is violated. Summer 906 sums each of the S_m $m=1:222$.

The operation of decision circuit 900' will now be explained in conjunction with the flow chart of Fig. 15. After an iteration of iterative decoding, the equation syndrome is calculated and summed by equal vector circuit 908 and summer 906. Threshold circuit 912 determines if the sum of all $S_m = 0$, then a signal is sent to multiplexer 854 to output $b_{c,k}$ from memory 904. If the sum of all $S_m \neq 0$, then a determination is made by threshold 910 whether the number of iterations is less than threshold value t_3 . If the number of iterations is less than threshold value t_3 , the next iteration is performed. Otherwise threshold circuit 912 determines if the sum of all S_m is less than threshold t_1 . If so, then a signal is sent to multiplexer 854 to output $b_{c,k}$ from memory 904. Alternatively, the next step in the process is to determine if b_k is involved in an equation that is violated. If b_k is not involved in an equation that is violated multiplexer 854 $b_{c,k}$ is provided as the output. Otherwise threshold 902 determines if $|U_rAPP_k|$ is less than threshold value (t_2), if so multiplexer 854 selects $b_{s,k}$ to be output. If $|U_rAPP_k|$ is not less than threshold value (t_2), multiplexer 854 selects $b_{c,k}$ to be output.

While the invention has been described in conjunction with several specific embodiments, it is evident to those skilled in the art that many further alternatives, modifications and variations will be apparent in light of the foregoing description. More specifically, while the present invention is preferably implemented as an integrated circuit, it is contemplated that the present invention may also be implemented as discrete components or a general-purpose processor operated in accordance with program code instructions or computer program or combination thereof. These program code instructions can be obtain from a medium, such as network, local area network, the Internet, or storage devices. Such storage devices include, by way of example, magnetic storage devices, optical storage devices, electronic storage devices, magneto-optical device and the like. Thus, the invention described herein is intended to embrace all such alternatives, modifications, applications and variations as may fall within the spirit and scope of the appended claims.